Moving into Third Space — High School Students' Funds of Knowledge in the Mathematics Classroom

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Students enter Australian high schools with a wide variety of knowledge, background experiences, attitudes to school and discourses, their "funds of knowledge". Much of this comes from their "first space" — what they bring from the home and community. Some of it comes from their "second space" — what they bring from prior school experiences and what they experience in the current school environment. This second space is profoundly shaped by the experience of formal, textbook-oriented approaches to high school mathematics. This theoretical paper looks at the experience of students entering their first year of high school mathematics in terms of first and second space, and casts forward to the creation of a "third space" — one which values and builds on students' funds of knowledge.

23 May 2002, 9.40am: Royal Avenue High School[1]

Lucy, a student teacher, is teaching a year 7 maths class. There are fifteen students in the class, mostly boys, clearly from a variety of cultural backgrounds. According to the teacher, Mr C, there are five students missing from today's lesson. Four of the students here today were absent yesterday. As they wait outside the class some students push each other so that they can get into the room quickly and have first choice of seat; one tells his friend about the fight at home the previous night; some talk loudly about how much being at school "sucks". When they enter the room many of the students race towards the single heater at the far side of the room, pushing over chairs or desks on their way. Lucy writes a few simple addition questions on the blackboard, and one or two of the students start to write some answers on paper. Another student yells that he doesn't have a pencil, others just sit there, others do not even seem to notice that there is anything written on the board. After a few minutes Lucy tries to talk to the class to check their answers, but no one seems to listen. Mr C seems powerless to help. The lesson proceeds, with Lucy handing out a blackline master with a mixture of addition and subtraction questions. It is all she can do to prevent the students from physically harming each other, vandalising the room, not climbing out of the windows, and not running out of the room and disturbing other classes.

23 May 2002, 11.30am: McWilliam College

Melinda, a student teacher, is teaching a year 7 maths class. There are twenty-six students in the class, about two thirds are girls, all dressed neatly in school uniforms. Ms L, their normal teacher arrives with Melinda to allow the students into the classroom. The room is arranged in a U-shape, and when they enter the room each student makes their way to their chair, puts their schoolbag on the floor, and takes out a pen, a textbook and an exercise book. Melinda writes some algebraic simplification questions on the board, and the

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[1] The names of the schools, teachers and student teachers in these vignettes have been changed. When I made these observations of the classes my focus was on the teaching skills of my student teachers, rather than the school students or the classroom environment. Hence the vignettes were written in hindsight. They are written as impressions, rather than as an attempt to record accurate details.

students all begin to write down their answers. When the students have marked their answers Melinda explains how to use the distributive law to expand brackets. The students proceed to complete twenty to thirty questions from their textbook, with Melinda and Ms L walking around the room helping.

Reflections of a University Supervisor

After observing these two lessons I asked myself what the agenda was in these two contrasting classroom environments. Was it about learning mathematics? Was it about learning to fit the school system? Why, for one group of students, did school "suck", while for the other it was a familiar and comfortable space. Why was one group openly defiant of the teacher, and the other compliant? As I thought more about the contrast, I could not help wondering which group of students had the more rational response to what they were being asked to do in the school mathematics classroom.

A Theoretical Framework for Examining the Discourse of the High School Mathematics Classroom

Learning mathematics in a high school classroom is much more than the acquisition of a set of skills and concepts described in curriculum frameworks or textbooks. It is a complex activity involving social interactions among twenty to thirty diverse individuals together enacting an unpredictable script whose goals may be as diverse as the actors themselves. In looking more closely at the events and interactions in a school mathematics classroom, it is essential to acknowledge the many different funds of knowledge, such as community, home and peer group, that shape the ways in which young people create mathematical meaning. Students are confronted, as they move from home to school to peer group and community, with the challenge of engaging in different Discourses (Gee, 2000; Moje et al., 2004) in different contexts. Learning and using mathematics within these different contexts may be foregrounded or marginalised, Discourses may conform to everyday or academic norms (Gee, 2000), and students find themselves at one time empowered within, and in the next minute alienated from, the home, school or community environment. The potential for alienation is particularly great as students make the transition from primary to high school, and face the challenge of creating their identity within a mathematics classroom that may take little account of their funds of knowledge. This paper presents a theoretical examination of how students' funds of knowledge play out in the mathematics classroom and casts forward to the creation of an environment that values and builds on the rich funds of knowledge brought by students as they enter high school mathematics.

First Space

Moll, Amanti, Neff and Gonzalez (1992) use the term 'funds of knowledge' to describe the historically and culturally accumulated bodies of knowledge and skills that enable people to function effectively as individuals and in society. In studies focused in local communities, they examined the impact of students' knowledge and understandings, and their values, attitudes and social interactions, on language and social studies, and developed a curriculum that valued and built on these funds of knowledge. They claimed that by building on these household resources they could make strategic connections between education and anthropology to develop a curriculum much richer than that often encountered in schools.

Gonzalez, Andrade, Civil, and Moll (2001) describe the BRIDGE project, in which they investigated the funds of knowledge found in Mexican American, African American and Native American communities. They endeavoured to uncover the mathematical potential of low income or disadvantaged households, and to look at the distributed nature of mathematical community capital. Having conducted ethnographic research into community practices and knowledge, they used teacher study groups to investigate effective pedagogical practices and develop curricular activities in the mathematics classroom that built on these funds of knowledge. They asked three critical questions: What counts as mathematics? How can we find mathematics within households that are economically marginal? How can we help parents and communities see themselves as mathematicians, "doing" mathematics in their everyday lives? In this way they addressed some of the gaps between the traditional curricular practices of the mathematics classroom and the out-of-school mathematical activities undertaken in communities.

Gonzalez, Andrade, Civil, and Moll (2001) built particularly on the work of Vygotsky, recognising the role of language in mediating thinking. Acknowledging that human beings and their social worlds are inseparable, and that human thinking is irreducible to individual properties or traits, Gonzalez, Andrade, Civil, and Moll (2001) reconceptualised their investigation of the mathematical knowledge of households from one which looked merely at conventional mathematical knowledge to one which studied a zone of practice incorporating knowledge, social relationships and mathematical activities.

Students in Australian schools, whether from disadvantaged communities or not, bring a rich background of knowledge, values, attitudes and patterns of social interaction, a 'first space' (Moje et al., 2004), from their home, community and peer networks. This first space is seldom tapped into or built on in the traditional high school mathematics classroom. Zevenbergen (2001) notes that the youth of today's society have been raised in a highly technological society, and bring to school different experiences and expectations from those brought by students in the past. She argues that the traditional practices of the high school mathematics classroom do not meet the needs of students, and that students can therefore become disengaged.

Second Space

Boaler (1998) studied students' experiences in mathematics at two very different schools, finding that in a traditional, streamed, textbook-oriented environment, students had lower self-concept, and achieved a less robust and transferable understanding of mathematical concepts than students in a more open, investigative mathematics classroom environment. The students in these two schools, then, occupied very different 'second spaces', ones which were more or less well attuned to their interests, their ways of learning and their patterns of relating to others. In turn, the students responded differentially, in both the academic and affective domains, to the Discourses of these second spaces. More recently Boaler (2003) has discussed the concept of agency in the mathematics classroom, arguing that in a traditional classroom environment students rely on the teacher or textbook as the source of knowledge and for confirmation of "right answers", rather than developing self-agency or looking to the agency of the discipline of mathematics. The locus of agency adopted by students in the mathematics classroom is, in large part, a response to the past and current second spaces in which they find themselves.

Gee (2000) considers that the second spaces of the classroom are predominantly places of Discourse, and that students often find themselves caught between familiar everyday discourses and more valued academic discourses. Gee (1989) uses the term Discourse (upper case D) to describe a combination of saying, doing, being valuing and believing. He claims that Discourses are not mastered by direct instruction, but are acquired by enculturation into social practices. Thus language, both the overt language of mathematics and the hidden messages contained within the language of the classroom, plays a crucial role in the construction of second space.

There is a long history of studies of language *in* mathematics. Zevenbergen (2000) discusses mathematics as a register with its own specialised vocabulary, semantic structure and lexical density. She maintains that language is a form of cultural capital, and suggests that students must learn to 'crack the code' of the mathematics classroom, a task which is less accessible to students from working class backgrounds than to students from middle class backgrounds.

A second emphasis, particularly in the constructivist literature, has been the role of language in *learning* mathematics. Brown (2001) points to the role of language in developing understanding, noting that mathematics can only be shared in discourse, mediated through social events. Alrø and Skovsmose (2002) stress that the quality of communication in the classroom is inextricably linked to the quality of learning, and discuss the role of dialogue in the learning of mathematics. In contrast to lessons containing genuine dialogue, many traditional mathematics lessons amount to little more than a game of 'guess what the teacher thinks' (Alrø & Skovsmose, 2002). In such a classroom students are often seen as empty receptors, and little if any attention is given to students' funds of knowledge.

A more recent area of focus on language in mathematics has been on language *and* mathematics, and in particular how language in the mathematics classroom illustrates power relationships (Walkerdine, 1988; Zevenbergen, 2001) and social practices. In a set of papers edited by Barwell (2005), bringing together the fields of mathematics education and applied linguistics, several writers considered issues such as the nature of academic mathematical discourse, and the relationship between the teaching and learning of mathematics and students' induction into mathematical discourses. They claim that a view of language as a social practice is inseparable from a view of mathematics as social practice. Every instance of communication in the mathematics classroom not only communicates mathematical concepts and relationships, but also interpersonal meanings, attitudes and beliefs (Morgan, 2006). Fairclough (1992) views every instance of discourse as simultaneously a piece of text, a discursive practice and a social practice.

Third Space

Hybridity theory (Bhabha, 1994) maintains that people make sense of their world through the integration of, and interactions between their multiple resources and funds of knowledge. It examines how being "in-between" may be both constraining and productive in one's identity development. Writing of her experiences as an artist of mixed race, Bolatigici (2004) sees being "in-between" as a potentially librating location of cultural resistance. This in-betweenness is termed 'third space' a hybrid space that brings together the sometimes competing and contradictory knowledges and discourses encountered in the physical and social environments between which people move.

While students in school may or may not share the in-betweenness of people of mixed race, they are in-between in many other ways. In particular, students who move from primary to high school are in-between childhood and adolescence; they are in-between home and the wider community as their preferred social space; they are in-between a high support and an often less supportive school environment; they are in-between informal and formal approaches to learning mathematics. The transition from primary to high school can be a challenging experience for many students. Feelings of alienation and disengagement are common as students move to an environment which may be seen to be less caring and supportive than the small primary school (Blyth, 1987). Factors which may assist or hinder students in making this transition include the influence of the peer group, support offered in the high school, the student's knowledge and beliefs about how past students made the transition, and whether the adolescent has a supportive family environment (Wallis & Barrett, 1998).

Mathematics, which is seen as a high status subject, can create particular feelings of anxiety (Dossel, 1993) as students make the transition to high school. These feelings can be exacerbated by a variety of factors, including competition, time limits, the language of the textbook and ability-grouping practices, in which students in the lower ability groups are given low-level questions, and often have the least qualified or competent teachers (Zevenbergen, 2002). Creating an in-between space in high school mathematics that is one of liberation and learning rather than oppression or conformity, as experienced by the students I observed at Royal Avenue High School and McWilliam College, is a significant and pressing challenge.

Moje, Ciechanowski, Kramer, Ellis, Carrillo and Collazo (2004) describe three perspectives on third space. A geographic perspective sees third space as arising from the binaries of everyday and academic discourses, of spontaneous and scientific concepts (Vygotsky, 1962) or of out-of-school or in-school ways of knowing or doing mathematics (Lave, 1988). A school mathematics classroom which takes account of this conception of third space might involve looking at how people use mathematics in the home or community, or at the ethnomathematics of different cultural groups (Borba, 1990). It would attempt to make explicit the similarities and differences between formal and informal ways of solving problems, with a focus on mathematics as it is used in the world. It would look specifically at situations in which the formal use of mathematics might help to solve problems, and at the limitations of formal mathematics. It might take account of and contribute to numeracy across the curriculum (Thornton & Hogan, 2005), valuing the less formal mathematical skills and ways of thinking used by students in other learning areas of the school.

A second perspective described by Moje et al. (2004) is a post-colonial one, questioning the privileged position of academic discourses. They argue that academic texts can limit students' learning as they seek to reconcile their own ways of knowing and communicating to those which are privileged in the classroom. Accepted classroom discourse, particularly in mathematics, is a historical product of a European world view, produced out of the colonisation and domination of the Other (Walkerdine, 1990). Thus students who do not progress according to the accepted model of developmental psychology arising from a rational, European view of the world are considered inferior, placed in the "bottom class", and given work which is of a lower intellectual quality than those who do progress (Wiliam & Bartholomew, 2005).

How might a post-colonial third space, that questions issues of domination and privilege, be created in the mathematics classroom? Frankenstein (2001) describes a critical mathematical literacy program, in which disengaged youth use mathematics to examine the social conditions in which they live, and use social issues to learn mathematics. By looking, for example, at how unemployment figures are constructed, students learn the mathematical concepts and skills of percentages and hence come to understand that the official figures are a political construction.

A third perspective described by Moje et al. (2004) is an educational one, which attempts to deal explicitly with providing the resources necessary for future social and cognitive development. Third space is seen as a bridge between home and community funds of knowledge and school-based discourse. It is a scaffold used to move students through zones of proximal development (Vygotsky, 1962) toward better academic knowledge.

Making the language of mathematics explicit is a crucial element in this educational third space, but a focus on mathematical vocabulary alone is inadequate. As Zevenbergen (2000) points out, mathematics has its own register, with distinctive and complex rules of grammar and syntax. The production of semiotic chains, in which learning moves from a consideration of everyday activity, through specific practice characterising that activity, to a model of that activity using a mathematical representation, and finally to a mathematical abstraction was found to be effective in scaffolding students' learning of mathematics in an undergraduate ethnomathematics course (Presmeg, 2006).

Parkin and Hayes (2006) also describe the use of scaffolding techniques to help students from Aboriginal backgrounds to access to language of word problems in mathematics textbooks. Using Gray's scaffolded literacy (Rose et al., 1999) stages of lower book orientation, higher book orientation, transformations, text patterning and short writes resulted in fewer teacher interventions, greater confidence and more frequent use of mathematical language in students' responses. In the context of the mathematics classroom, lower book orientation, or building the field, consisted of asking students why the writers of textbooks used word problems and what the function of word problems was. Higher book orientation consisted of analysing the functions of particular groups of words and the language choices that the author made to realise those functions. Transformations consisted of moving the words from their original place into a mode where they could be easily manipulated, and included an illustration of the problem. Text patterning involved students in creating a new grammatical pattern by taking away the author's original word choices. Finally, short writes involved students in constructing their own related word problems, which were of relevance and interest to them. While this project focused specifically on word problems as the teachers involved identified this as an important aspect of students' transition to high school, similar scaffolding techniques could be used productively to provide students with greater access to all formal mathematical writing, such as is commonly found in school textbooks.

Conclusion: Applying Third Space to the Mathematics Classroom

A focus on students' funds of knowledge provides a promising lens with which to view discourse in the mathematics classroom. The possibility of a mismatch between students' home and community funds of knowledge (first space) and their school funds of knowledge (second space) has traditionally been viewed as a problem to be overcome, and is often the reason for grouping students into academically proficient or deficient classes. The students at Royal Avenue High School were placed in a low level mathematics class, ostensibly because they struggled to understand mathematics, but in fact because they did not fit the expected behaviours and discourses of the school. The students at McWilliam College were in the top level mathematics class, ostensibly because they were mathematically able, but they were viewed as able because they were content to listen to the teacher and compliantly complete textbook questions. It could be argued that the students' defiant or compliant responses to the diet of mathematics being offered in the classrooms as described in the vignettes were unproductive in both schools.

Rather than seeing the mismatch as a problem, third space sees both students' home and community funds of knowledge and their school funds of knowledge as a resource through which to empower students as effective learners in the school situation. The three perspectives on third space described by Moje et al. (2004) have the potential to inform an inclusive curriculum and pedagogy that is rich in cultural and practical relevance, that values what students bring to the classroom, and that promotes high levels of mathematical thinking.

For the students at the schools described in the vignettes these third spaces will include experiences that seek to explore and build on students' informal ways of solving problems, that examine the cultural roots of the mathematics studied in schools, that use mathematics to look at issues of critical importance to students, and that are carefully scaffolded to induct students into the academic discourse of the mathematics community.

Asking the three questions (Gonzalez et al., 2001): "What counts as mathematics? How can we find mathematics within households that are economically marginal? How can we help parents and communities see themselves as mathematicians, "doing" mathematics in their everyday lives?" could lead to dramatically different mathematical experiences for each of these groups of students. For the students at Royal Avenue High School being able to see mathematics as something that they have been doing all of their lives, as something that happens in the community, and as something that helps them to understand how the world works, may well help them to find a hybrid third space in which their knowledge is valued and built on by the mathematics they experience in school. For the students at McWilliam College seeing mathematics as a way of thinking that is beyond textbook exercises and as a discourse within which you can ask questions and challenge existing knowledge and beliefs may well create a hybrid space that is more meaningful and productive than the teacher-directed, compliant space which they currently inhabit. Growing third space is crucial to promoting active, engaged and meaningful school mathematics classrooms.

References

Alrø, H., & Skovsmose, O. (2002). *Dialogue and learning in mathematics education: Intention, reflection, critique*. Dordrecht: Kluwer.

Barwell, R. (2005). Language in the mathematics classroom. *Language and Education, 19*(2), 97-101.

Bhabha, H. (1994). *The location of culture*. New York: Routledge.

Blyth, D. A. (1987). The impact of cumulative change in early adolescence. *Child Development, 58*(5), 1220.

Boaler, J. (1998). Open and closed mathematics: Student experiences and understandings. *Journal for Research in Mathematics Education, 29*(1), 41-62.

Boaler, J. (2003). *Studying and capturing the complexity of practice - the case of the 'dance of agency'.* In N. Pateman, B. Dougherty and J. Zilliox (Eds). Navigating Between Theory and Practice. Proceedings of the 2003 Joint Meeting of PME and PMENA, Honolulu, HI.

Bolatigici, T. (2004). Claiming the (N)either/(N)or of 'third space': (re)presenting hybrid identity and the embodiment of mixed race. *Journal of Intercultural Studies, 25*(1), 75-85.

Borba, M. (1990). Ethnomathematics and education. *For the Learning of Mathematics, 10*(1), 39-43.

Brown, T. (2001). *Mathematics education and language: Interpreting hermeneutics and post-structuralism.* Dordrecht: Kluwer.

Dossel, S. (1993). Maths anxiety. *Australian Mathematics Teacher, 49*(1), 4-8.

Fairclough, N. (1992). *Discourse and social change*. Cambridge, UK: Polity Press.

Frankenstein, M. (2001). *Reading the world with math: Goals for a criticalmathematical literacy curriculum.* In Mathematics: Shaping Australia. Proceedings of the 16th Biennial Conference of the Australian Association of Mathematics Teachers Inc., Canberra. 53-64

Gee, J. P. (1989). Literacy, discourse and linguistics: Introduction. *Journal of Education, 171*(1), 5-17.

Gee, J. P. (2000). Teenagers in new times: A new literacy studies perspective. *Journal of Adolescent and Adult Literacy, 43*(5), 412-420.

Gonzalez, N., Andrade, R., Civil, M., & Moll, L. (2001). Bridging funds of distributed knowledge: Creating zones of practices in mathematics. *Journal of Education for Students Placed at Risk, 6*(1/2), 115-132.

Lave, J. (1988). *Cognition in practice*. Cambridge, MA: The University Press.

Moje, E., Ciechanowski, K., Kramer, K., Ellis, L., Carrillo, R., & Collazo, T. (2004). Working toward third space in content area literacy: An examination of everyday funds of knowledge and discourse. *Reading Research Quarterly, 39*(1), 38-70.

Moll, L. C., Amanti, C., Neff, D., & Gonzalez, N. (1992). Funds of knowledge for teaching: Using a qualitative approach to connect homes and classrooms. *Theory Into Practice, 31*(2), 132.

Morgan, C. (2006). What does social semiotics have to offer mathematics education research? *Educational Studies in Mathematics, 61*(1).

Parkin, B., & Hayes, J. (2006). Scaffolding the language of maths. *Literacy Learning: the Middle Years, 14*(1), 23-35.

Presmeg, N. (2006). Semiotics and the "connections" standard: Significance of semiotics for teachers of mathematics. *Educational Studies in Mathematics, 61*(1).

Rose, D., Gray, B., & Cowey, W. (1999). Scaffolding reading and writing for indigenous children in school. In P. Wignell (Ed.), *Double power: English literacy and indigenous education*. Melbourne: Language Australia.

Thornton, S., & Hogan, J. (2005). *Mathematics for everybody: Implications for the lower secondary school.* In M. Coupland, J. Anderson and T. Spencer (Eds). Making Mathematics Vital. Proceedings of the 20th Biennial Conference of the Australian Association of Mathematics Teachers Inc., Sydney. 243-252

Vygotsky, L. (1962). *Thought and language*. Cambridge: Massachusetts Institute of Technology Press.

Walkerdine, V. (1988). *The mastery of reason*. London: Routledge.

Walkerdine, V. (1990). Difference, cognition and mathematics education. *For the Learning of Mathematics, 10*(3), 51-56.

Wallis, J. R., & Barrett, P. M. (1998). Adolescent adjustment and the transition to high school. *Journal of Child & Family Studies, 7*(1), 43-58.

Wiliam, D., & Bartholomew, H. (2005). It's not which school but which set you're in that matters: the influence of ability grouping practices on student progress in mathematics. *British Educational Research Journal*. 30(2) 279-293

Zevenbergen, R. (2000). "Cracking the code" of mathematics classrooms: School success as a function of linguistic, social, and cultural background. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 201-224). Westport, CT: Ablex.

Zevenbergen, R. (2004). Reconceptualising numeracy for new times. *Curriculum Perspectives 24*(3) 1-7

Zevenbergen, R. (2002). To stream or not to stream: Students' experiences of ability grouping in mathematics. *Teaching Mathematics, 27*(3), 4-10.